NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

# FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
## SCHOOL OF NATURAL AND APPLIED SCIENCES
### DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

| | |
|---|---|
| **QUALIFICATION:** BACHELOR OF SCIENCES IN APPLIED MATHEMATICS AND STATISTICS | |
| **QUALIFICATION CODE:** 07BAMS | **LEVEL:** 7 |
| **COURSE CODE:** SMS701S | **COURSE:** SURVEY METHODS AND SAMPLING TECHNIQUES |
| **SESSION:** JULY 2023 | **PAPER :** THEORY |
| **DURATION:** 3 Hours | **MARKS:** 100 |

| **SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER** | |
|---|---|
| **EXAMINER** | Mr. J. J. SWARTZ |
| **MODERATOR:** | Dr. I. NEEMA |

## INSTRUCTIONS

1. Answer all the questions in the booklet provided
2. Show clearly all the steps used in the calculations.
3. Write clearly and neatly.
4. Number the answers clearly.

## PERMISSIBLE MATERIALS

1. Calculator

## ATTACHMENTS

1. Normal distribution table
2. T-table
3. Chi-square table

**THIS QUESTION PAPER CONSISTS OF 5 PAGES** (Including this front page)

**Question 1 [25 marks]**

1.1 Provide a diagrammatically representation of a survey from a process perspective. [6]

1.2. Which academic discipline provides the framework for understanding how interviewer behaviours may influence the activities of respondents, both when they are recruited as respondents and during the survey interview? [2]

A. Mathematics
B. Social science
C. Social psychology
D. Computer science

1.3. Which inferential steps are central to the answers people give that accurately describe characteristics of the respondents. [3]

A. Inferences made from the respondents answer to questions to the characteristics of the respondent
B. Inferences made from the characteristics of the sample to the characteristics of the population

1.4. In the 2015 Namibia Income and Expenditure survey, the following data was collected from households. Write the appropriate variable type for each one of the variables and indicate what type of estimates that can be produced from the data.

1.4.1 Total monthly household expenditure [2]
1.4.2 Age in completed years of household heads [2]
1.4.3 Marital status of household heads [2]

1.5. A questionnaire is a data collection tool used to collect data in all survey-based studies. Please state the three important sections in the structure of the questionnaire and elaborate on the type of information collected in the information section. [4]

1.6. Personal interviews are one of the approaches of gathering survey data, provide two advantages and two disadvantages of personal interviews. [4]

**Question 2 [25 marks]**

2.1 State the four major sampling designs in probability sampling? [4]

2.2 The Namibia Statistics Agency (NSA) wants to estimate the rate of incidence of respiratory disorders among the middle aged male and female smokers in Namibia. How large a sample should be taken to be 95% confident that the error of estimation of the proportion of the population with such disorders does not exceed 0.05? The true value of p is expected to be near 0.20. [4]

2.3 Surveys, which cover most real life situations, are multi-purpose. Thus, units within a stratum may be alike for certain major characteristics but may be very different for other characteristics. In

such situations strata must be formed with primary interest on major survey characteristics. Provide three other reasons for stratification. [3]

2.4 The following data was collected from a random sample of 20 households of a certain community consisting of 250 households. Assume the population distributions are close to a normal distribution.

**Table 1: Sample data**

| Household ID | Sex of head: 1=Female, 2=Male | Household size | Monthly household income (N$) |
|---|---|---|---|
| 1 | 1 | 8 | 150 |
| 2 | 2 | 2 | 245 |
| 3 | 2 | 3 | 450 |
| 4 | 1 | 5 | 120 |
| 5 | 2 | 4 | 300 |
| 6 | 2 | 5 | 200 |
| 7 | 2 | 1 | 500 |
| 8 | 1 | 7 | 175 |
| 9 | 2 | 3 | 275 |
| 10 | 1 | 10 | 200 |
| 11 | 2 | 2 | 250 |
| 12 | 1 | 3 | 550 |
| 13 | 1 | 2 | 500 |
| 14 | 2 | 1 | 230 |
| 15 | 2 | 2 | 250 |
| 16 | 2 | 5 | 580 |
| 17 | 2 | 6 | 600 |
| 18 | 1 | 2 | 350 |
| 19 | 2 | 1 | 450 |
| 20 | 1 | 3 | 500 |

2.4.1 Compute the estimates for the proportions (in %) of male and female headed households in this community. [2]

2.4.2 Calculate estimates for the average household size of the female and male headed households in this community. [2]

2.4.3 Estimate the total monthly income of this community based on the sample data. [3]

2.4.4 Calculate the standard error of the estimated total monthly income in (2.4.3) [3]

2.4.5 Construct a 95% confidence interval for the total monthly income of this population [3]

2.4.6 Comment about the monthly incomes of the female and male headed households. [1]

**Question 3 [25 marks]**

3.1 Data can be classified by its scales of measurement. State the four scales of measurements and explain the difference between them. [4]

3.2 State the properties of estimators and illustrate the property **unbiasedness** of an estimator using the following information: A population consists of N = 6 HH's. Select a sample of n = 2 HH's to estimate the average HH size and the total number of persons in the households in the population. [11]

| Population unit – Ui | Household size – Yi |
|---|---|
| U1 | 1 |
| U2 | 3 |
| U3 | 4 |
| U4 | 5 |
| U5 | 4 |
| U6 | 1 |

3.3 Select a random sample of 10 elements from the following list using the random number table given below:

**These serial numbers indicate a list of 20 households:**

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

**Random number table:**

55 09 79 15 11 56 65 88 08 16 96 95 33 17 60 45 81 31 50 46 79 19 16
49 99 08 80 01 56 35 41 42 72 58 20 39 33 53 85 26 [4]

3.4 A population consists of N = 5,000 persons. A Simple Random Sample, without replacement of n = 50 included 10 persons of Khoisan descent.

3.4.1 Calculate a 95% confidence interval for P, the proportion of persons of Khoisan descent in the population. [3]

3.4.2 Suppose we would like to estimate P the proportion of persons of Khoisan descent to within $\pm 3\%$ with 95% confidence. What sample size is necessary? (Assume P to be 0.5). [3]

**Question 4 [25 marks]**

4.1 The two common forms of scanning techniques are the optical character reader (OCR) and optical mark reader (OMR). What are the advantages of OMR systems over other types of data entry, particularly where time and accuracy are important? [5]

4.2 You have been approached by a client, Ministry of Health and Social Services to process the 2013 DHS PLUS data and are required to estimate the number of working days you will take to enter 10, 000 questionnaires.

Assumptions:

(i) **Ten** percent of the equipment may not be operational at any point in time because of mechanical breakdown or operator absence.

(ii) **Five** percent of the data will have to be rekeyed because of errors encountered in verification.

(iii) Keying of manual corrections during editing will be the equivalent of **five** percent of the original workload.

The following information is provided:

- 10 x data entry stations
- 2 x shifts of data entry operators
- 7 x productive hours per work
- 10 x operators

- Average of 8,000 strokes per hour
- 10,000 questionnaires
- 2,500 strokes per questionnaires
- 100 percent verification

[10]

4.3 You were asked by a client to analyze the 2009/10 Namibia Household Income and Expenditure Survey data. The following table and figure were produced from the data. Please write a short narrative or interpretation for the tables below on what they are depicting.

**4.3.1 Table 2: Dependency ratios for 2008 and 2012**

| Age group | 2008 | | 2012 | |
|---|---|---|---|---|
| | Number | Dependency ratio | Number | Dependency ratio |
| 0 - 14 | 682 286 | 71.1 | 767 557 | 64.4 |
| 65+ | 102 614 | 10.7 | 106 904 | 9.0 |
| Total | 784 900 | 81.8 | 874 461 | 73.4 |

[5]

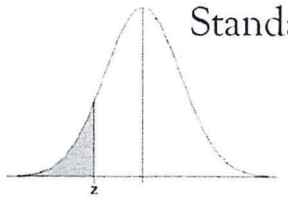
**4.3.2 Table 3: Population by sex and age group**

| Age group | Female | | Male | | Both sexes | | Sex ratio |
|---|---|---|---|---|---|---|---|
| | Number | % | Number | % | Number | % | |
| 00-04 | 139 287 | 12.9 | 135 161 | 13.8 | 274 520 | 13.3 | 97.0 |
| 05-09 | 125 157 | 11.6 | 124 931 | 12.7 | 250 159 | 12.1 | 99.8 |
| 10-14 | 122 151 | 11.3 | 121 727 | 12.4 | 243 878 | 11.8 | 99.7 |
| 15-19 | 128 831 | 11.9 | 120 609 | 12.3 | 249 440 | 12.1 | 93.6 |
| 20-24 | 108 224 | 10.0 | 97 720 | 9.9 | 206 016 | 10.0 | 90.3 |
| 25-29 | 89 582 | 8.3 | 77 201 | 7.9 | 166 783 | 8.1 | 86.2 |
| 30-34 | 74 899 | 6.9 | 67 550 | 6.9 | 142 449 | 6.9 | 90.2 |
| 35-39 | 59 482 | 5.5 | 55 844 | 5.7 | 115 326 | 5.6 | 93.9 |
| 40-44 | 51 240 | 4.7 | 39 868 | 4.1 | 91 108 | 4.4 | 77.8 |
| 45-49 | 42 182 | 3.9 | 34 276 | 3.5 | 76 457 | 3.7 | 81.3 |
| 50-54 | 32 321 | 3.0 | 28 161 | 2.9 | 60 482 | 2.9 | 87.1 |
| 55-59 | 25 720 | 2.4 | 21 223 | 2.2 | 46 943 | 2.3 | 82.5 |
| 60-64 | 21 586 | 2.0 | 17 514 | 1.8 | 39 100 | 1.9 | 81.1 |
| 65-69 | 16 662 | 1.5 | 13 154 | 1.3 | 29 816 | 1.4 | 78.9 |
| 70-74 | 13 370 | 1.2 | 9 286 | 0.9 | 22 656 | 1.1 | 69.5 |
| 75-79 | 10 923 | 1.0 | 7 735 | 0.8 | 18 658 | 0.9 | 70.8 |
| 80-84 | 8 576 | 0.8 | 4 239 | 0.4 | 12 815 | 0.6 | 49.4 |
| 85-89 | 6 326 | 0.6 | 2 261 | 0.2 | 8 588 | 0.4 | 35.7 |
| 90-94 | 2 902 | 0.3 | 1 209 | 0.1 | 4 110 | 0.2 | 41.7 |
| 95+ | 2 122 | 0.2 | 709 | 0.1 | 2 831 | 0.1 | 33.4 |
| Not Stated | 1 805 | 0.2 | 2 458 | 0.3 | 4 263 | 0.2 | 136.2 |
| **Total** | **1 083 347** | **100** | **982 836** | **100** | **2 066 398** | **100** | **90.7** |

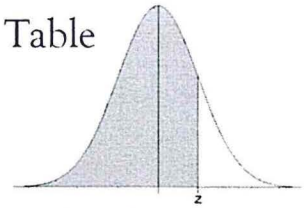
[5]

**********END OF EXAMINATION! **********

# Standard Normal Distribution Probabilities Table




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| −0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| −0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| −0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| −0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| −0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

**Confidence Interval Critical Values, $z_{\alpha/2}$**

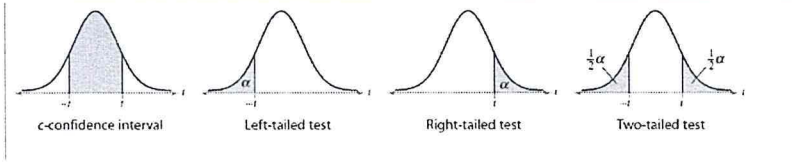
| Level of Confidence | Critical Value, $z_{\alpha/2}$ |
|---|---|
| 0.90 or 90% | 1.645 |
| 0.95 or 95% | 1.96 |
| 0.98 or 98% | 2.33 |
| 0.99 or 99% | 2.575 |

**Hypothesis Testing Critical Values**

| Level of Significance, $\alpha$ | Left-Tailed | Right-Tailed | Two-Tailed |
|---|---|---|---|
| 0.10 | − 1.28 | 1.28 | ±1.645 |
| 0.05 | − 1.645 | 1.645 | ±1.96 |
| 0.01 | − 2.33 | 2.33 | ±2.575 |

# Student t Distribution Probabilities Table

| one-tail area | 0.25 | 0.125 | 0.1 | 0.075 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
|---|---|---|---|---|---|---|---|---|---|
| two-tail area | 0.5 | 0.25 | 0.2 | 0.15 | 0.1 | 0.05 | 0.02 | 0.01 | 0.001 |
| confidence level | 0.5 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 0.98 | 0.99 | 0.999 |
| d.f. 1 | 1.000 | 2.414 | 3.078 | 4.165 | 6.314 | 12.706 | 31.821 | 63.657 | 636.619 |
| 2 | 0.816 | 1.604 | 1.886 | 2.282 | 2.920 | 4.303 | 6.965 | 9.925 | 31.599 |
| 3 | 0.765 | 1.423 | 1.638 | 1.924 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 |
| 4 | 0.741 | 1.344 | 1.533 | 1.778 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 0.727 | 1.301 | 1.476 | 1.699 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 |
| 6 | 0.718 | 1.273 | 1.440 | 1.650 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 |
| 7 | 0.711 | 1.254 | 1.415 | 1.617 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 |
| 8 | 0.706 | 1.240 | 1.397 | 1.592 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 0.703 | 1.230 | 1.383 | 1.574 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 0.700 | 1.221 | 1.372 | 1.559 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 0.697 | 1.214 | 1.363 | 1.548 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 0.695 | 1.209 | 1.356 | 1.538 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 0.694 | 1.204 | 1.350 | 1.530 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 0.692 | 1.200 | 1.345 | 1.523 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 0.691 | 1.197 | 1.341 | 1.517 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 0.690 | 1.194 | 1.337 | 1.512 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 0.689 | 1.191 | 1.333 | 1.508 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 0.688 | 1.189 | 1.330 | 1.504 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 0.688 | 1.187 | 1.328 | 1.500 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 0.687 | 1.185 | 1.325 | 1.497 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 0.686 | 1.183 | 1.323 | 1.494 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 0.686 | 1.182 | 1.321 | 1.492 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 0.685 | 1.180 | 1.319 | 1.489 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 0.685 | 1.179 | 1.318 | 1.487 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 0.684 | 1.198 | 1.316 | 1.485 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 0.684 | 1.177 | 1.315 | 1.483 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 0.684 | 1.176 | 1.314 | 1.482 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 0.683 | 1.175 | 1.313 | 1.480 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 0.683 | 1.174 | 1.311 | 1.479 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 0.683 | 1.173 | 1.310 | 1.477 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 35 | 0.682 | 1.170 | 1.306 | 1.472 | 1.690 | 2.030 | 2.438 | 2.724 | 3.591 |
| 40 | 0.681 | 1.167 | 1.303 | 1.468 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 |
| 45 | 0.680 | 1.165 | 1.301 | 1.465 | 1.679 | 2.014 | 2.412 | 2.690 | 3.520 |
| 50 | 0.679 | 1.164 | 1.299 | 1.462 | 1.676 | 2.009 | 2.403 | 2.678 | 3.496 |
| 60 | 0.679 | 1.162 | 1.296 | 1.458 | 1.671 | 2.000 | 2.390 | 2.660 | 3.460 |
| 70 | 0.678 | 1.160 | 1.294 | 1.456 | 1.667 | 1.994 | 2.381 | 2.648 | 3.435 |
| 80 | 0.678 | 1.159 | 1.292 | 1.453 | 1.664 | 1.990 | 2.374 | 2.639 | 3.416 |
| 100 | 0.677 | 1.157 | 1.290 | 1.451 | 1.660 | 1.984 | 2.364 | 2.626 | 3.390 |
| 500 | 0.675 | 1.152 | 1.283 | 1.442 | 1.648 | 1.965 | 2.334 | 2.586 | 3.310 |
| 1000 | 0.675 | 1.151 | 1.282 | 1.441 | 1.646 | 1.962 | 2.330 | 2.581 | 3.300 |
| infinity | 0.674 | 1.150 | 1.282 | 1.440 | 1.645 | 1.960 | 2.326 | 2.576 | 3.291 |




# Chi Squared ($\chi^2$) Distribution Probabilities

Area to the Right of Critical Value

| d.f. | 0.995 | 0.99 | 0.975 | 0.95 | 0.9 | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | — | — | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |